

2. 10. 1

$$T_A e_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

同様に.

$$T_A e_2 = \begin{pmatrix} b \\ d \end{pmatrix}, \quad T_B e_1 = \begin{pmatrix} \alpha \\ r \end{pmatrix}, \quad T_B e_2 = \begin{pmatrix} \beta \\ s \end{pmatrix}$$

$$d.7. \quad T_A \otimes T_B (e_1 \otimes e_1) = \begin{pmatrix} a \\ c \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ r \end{pmatrix}$$

$$= \left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \right) \otimes \left\{ \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ r \end{pmatrix} \right\}$$

~~計算を省略する~~

~~計算~~

$$= \begin{pmatrix} a \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ r \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \otimes \begin{pmatrix} 0 \\ r \end{pmatrix}$$

$$= a\alpha e_1 \otimes e_1 + ar(e_1 \otimes e_2) + c\alpha(e_2 \otimes e_1) + cr(e_2 \otimes e_2)$$

$$\text{同様に } T_A \otimes T_B (e_1 \otimes e_2) = \begin{pmatrix} a \\ c \end{pmatrix} \otimes \begin{pmatrix} \beta \\ s \end{pmatrix}$$

$$= a\beta e_1 \otimes e_1 + as e_1 \otimes e_2 + c\beta e_2 \otimes e_1 + cs e_2 \otimes e_2$$

$$T_A \otimes T_B (e_2 \otimes e_1) = \begin{pmatrix} b \\ d \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ r \end{pmatrix}$$

$$= b\alpha e_1 \otimes e_1 + br e_1 \otimes e_2 + d\alpha e_2 \otimes e_1 + dr e_2 \otimes e_2$$

$$T_A \otimes T_B (e_2 \otimes e_2) = \begin{pmatrix} b \\ d \end{pmatrix} \otimes \begin{pmatrix} \beta \\ s \end{pmatrix}$$

$$= b\beta e_1 \otimes e_1 + bs e_1 \otimes e_2 + d\beta e_2 \otimes e_1 + ds e_2 \otimes e_2$$

以上を表現行列に

$$\begin{pmatrix} a\alpha & a\beta & b\alpha & b\beta \\ ar & as & br & bs \\ c\alpha & c\beta & d\alpha & d\beta \\ cr & cs & dr & ds \end{pmatrix}$$